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LETTER TO THE EDITOR

A new algorithm to extract the backbone in a random resistor network

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Abstract. We present an efficient algorithm to determine the backbone (the current-carrying part of a lattice) in the random resistor network problem. It reduces mainly to the determination of the cluster structure on two lattices. Its efficiency is due to the use of the dual lattice and, therefore, is specific to the two-dimensional case.

The geometrical structure of the current-carrying part, the 'backbone', of the random resistor network [1] at the percolation threshold is of great interest, both *per se* and for speeding up numerical studies of DC transport properties which only take place on this substructure (like, for example, the current distribution). However this task is generally time consuming [2] and it has even been suggested recently [3] that an efficient way of solving the current distribution in the lattice (namely a 'Fourier accelerated conjugate gradient' algorithm) could be more effective to extract the backbone than other methods which are based on a purely geometric approach.

To our knowledge, two geometric algorithms have been proposed to extract the backbone [2, 4]. The first one [2] consists mainly in 'burning' part of the structure a large number of times (basically, as many times as there are loops in the cluster) and progressively adding new elements to a substructure of the lattice until the backbone is complete. This algorithm is by no means easy to implement on a computer and requires a lot of computation time. The second one [4] makes full use of the specificities of the LISP language; however, the underlying algorithm is similar to the one described in the previous method. We propose here a new and simple method which mainly consists in identifying the complete cluster distribution on two different lattices. Moreover this program is easy to write in any language and fast to run. The computation time needed is basically twice that needed for the identification of clusters. Thus the time needed is proportional to the number of sites.

We consider a lattice made out of resistors and where a fraction $1-p$ of the bonds are removed at random. The lattice is placed between two bus-bars which act as equipotential electrodes. Now, there are two possible definitions of the backbone that one can use.

(i) The effective backbone. It is the set of bonds in which flows a non-zero current when a voltage is applied across the electrodes.

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(ii) The geometric backbone (see figure 1). A bond is part of this structure if there exist two non-intersecting paths which connect one end of the bond to one electrode and the other end to the other electrode.

The only difference between these two possible definitions is the set of perfectly balanced bonds (like a Wheatstone bridge). It is often assumed that they represent only a negligible part of the backbone. However a recent study [5] suggests, on the contrary, that they have a fractal dimension equal to that of the backbone itself, and therefore that their density is constant (about 0.159% for bond percolation on a square lattice).

The algorithm we propose here requires three steps. The last two demand the identification of connected clusters. Let us first of all assume that this task is easily performed on any lattice (for an efficient algorithm see [6]). For simplicity we will also consider first the case of a lattice of coordination number 3 (e.g. honeycomb).

The first step consists in identifying the incipient infinite cluster. In other words, we remove all clusters which are isolated from the electrodes and thus do not participate in the backbone; moreover, we make sure at the same time that the lattice we consider is connected (i.e. conducting). This basic task can be efficiently performed by a single 'burning' of the lattice [2].

The second step deals with the dual lattice (see figure 2): to each cell of the primal (original) lattice we associate a site in the dual network. Each bond of the dual lattice crosses one and only one bond in the primal one. (Hence the square lattice is self-dual, the honeycomb lattice is dual to the triangular lattice, etc.) Now each present bond is associated with a missing bond in the dual lattice and conversely a missing bond is associated with a present dual one. Let us note now that a deadend (in the original problem) is connected to the backbone by a single bond (since the coordination of the lattice is three). Therefore the deadend will be surrounded by a set of missing bonds in the primal lattice. Thus, the connection bond of the deadend to the backbone is associated in the dual lattice with a missing bond that is between two sites which belong to the same cluster of connected sites (such as between A and B in figure 2). In the second step, we begin by identifying connected clusters in the dual lattice. Then, we sweep through the primal lattice and remove all bonds that are in between two neighbouring cells connected in the dual lattice. After this second step, the sites that

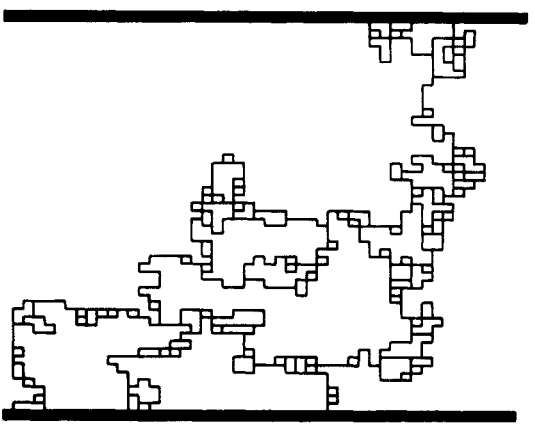

Figure 1. An example of a geometric backbone extracted from a 50×50 square lattice at the percolation threshold with the algorithm described in this letter.

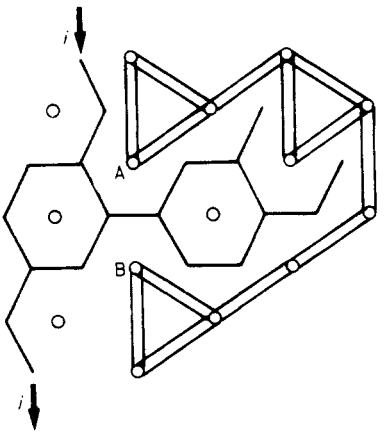

Figure 2. When some current i flows in this honeycomb lattice as indicated by the arrows, the structure located on the right-hand side of A—B is a dead-arm. We can identify it by considering the dual (triangular) lattice whose sites are indicated by circles and bonds by a double line, where A and B belong to the same dual cluster. The spirit of the method developed here is to cut all bonds similar to the one in between A and B.

used to be in the incipient infinite cluster after the first step are now either on the backbone or belong to disconnected clusters.

It is finally straightforward to conclude the computation in a third step by extracting, for instance again through burning, the incipient cluster which is now the backbone of the original random resistor network.

Depending on the algorithm one has used to find the incipient infinite cluster in the first step, it could happen that some clusters are left if they are connected to only one electrode by more than one connection (e.g. the cluster located between A and B in figure 3). They would actually carry a non-zero current if the electrode had a finite resistance. In the case of 'superconducting' electrodes, as is the case here, one should remove these structures. This can be achieved in the following way. When the second

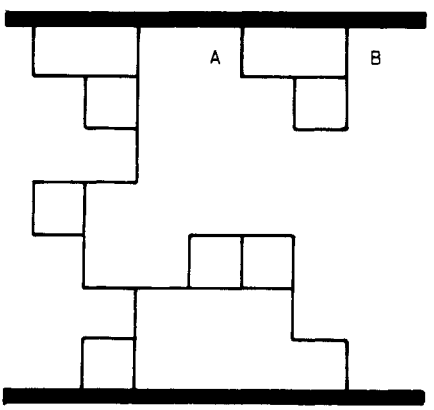

Figure 3. The crude version of the algorithm presented in figure 2 will leave some clusters attached to the electrode such as the one in between A—B. We can, however, remove these clusters in cutting all bonds, attached to the electrode, which are located in between two sites A and B belonging to the same cluster in the dual lattice.

step has been performed, we follow the row of cells (sites of the dual lattice) which are neighbours of one electrode. Whenever we encounter two sites that belong to the same cluster in the dual lattice, it means that this cluster surrounds the set of bonds connected to the electrode somewhere between the cells. Thus they cannot be part of the backbone. So we cut all bonds between these two cells. This quick operation is enough to get rid of all these clusters.

We can now extract the backbone on any lattice of coordination number three. Larger connectivities give rise to an additional difficulty: loops can be connected to the backbone at one node (see figure 4(a)). In this case, they will not be removed by the method presented above. (If one is only interested in universal properties of backbones, these structures can be left in the lattices, since it is possible to find some lattice on which they would carry a non-zero current (for instance, one can replace the nodes of the lattice by a set of resistive bonds that would always be present and such that the topology of the lattice is kept unchanged (see figure 4(b))). Universality then results in having the same scaling properties for the backbone with or without these 'dangling loops'. On the other hand, if one needs the exact backbone for another purpose, then these loops should be removed.) To remove them, we propose the following algorithm. Just after the second step, i.e. after the singly connected deadends have been removed one can identify the sites, S, at which deadloops, like that shown in figure 4(a), are attached. They all have a connectivity of at least four present bonds and, in addition, there exist two diagonally opposed sites on the dual lattice, say A and B, neighbouring S which belong to the same cluster in the dual lattice. Each site S that has been identified is now artificially split it into two sites, S′ and S″, as shown in figure 4(c) and the bond between S′ and S″ is not present. So one actually has a lattice with a slightly different topology: S′ and S″ have coordination three. In this way the dangling loop and the backbone have been disconnected, but one still has to

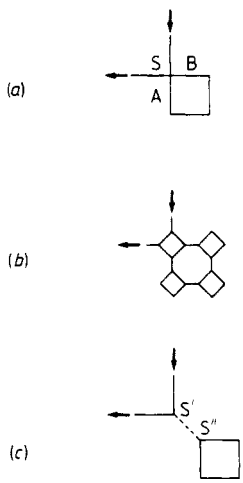

Figure 4. In a lattice of coordination number larger than 3, we will find some structures such as this dangling loop attached to the backbone in S (a). It would, however, be part of the backbone if we turn the junction nodes of this square lattice into small squares where all bonds are always present (b). Therefore, if one is interested in the scaling properties of backbones, one can keep these dangling loops. On the contrary, if one wants the effective backbone of this square lattice, one needs to artificially split these nodes in two sites S′ and S″ with a missing bond in between (broken line in (c)).

identify which of the two pieces is the backbone and which is the dangling loop. For this purpose one does another burning (the third step) after all sites S have been identified and separated in the above manner. Automatically, the disconnected loops will not be burned and discarded. This trick works for any coordination number larger than three, including the case of a triangular lattice (coordination number of six) in which two loops can be connected to the backbone at one single site.

We have obtained an algorithm that extracts the geometric backbone from any lattice in two dimensions. The most time-consuming operation is the identification of clusters. The latter problem is very classical and different algorithms are known to handle it. A very efficient one has been proposed by Hoshen and Kopelman [6].

Let us mention that this method makes full use of the properties of the dual lattice, and therefore cannot be extended to the case of a dimension of space larger than two. In this case, the algorithms of Herrmann *et al* [2] or Jan [4] are to be used.

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References

[1] Shlifer G, Klein W, Reynolds P J and Stanley H E 1979 *J. Phys. A: Math. Gen.* **12** L169
[2] Herrmann H J, Hong D C and Stanley H E 1984 *J. Phys. A: Math. Gen.* **17** L261
[3] Batrouni G G, Hansen A and Nelkin M 1986 *Phys. Rev. Lett.* **57** 1336
Batrouni G G and Hansen A 1987 *Preprint*
[4] Jan N 1987 private communication
[5] Batrouni G G, Hansen A and Roux S 1987 *Preprint*
[6] Hoshen J and Kopelman R 1976 *Phys. Rev.* B **14** 3438